space which does not contain any such extraneous feature. But also, viewed from the other side, in extensional analysis of this type appropriate to isotropic Euclidean manifolds, the vector operator ∇ and its powers are the fundamental ones, so that it is not really very surprising that gravitational analysis can be linked up with a theory of deformable space.

Finally and again, this re-statement of theories of relativity as relations of correspondence in space and time, by aid of uniform auxiliary manifolds of higher dimensions, may appear retrograde: in the earliest phase relativity was just such correspondence.* But it has the advantage of getting rid of the very puzzling auxiliary apparatus of local timekeepers, and their changes of rate when moved about. And, moreover, it is not, in fact, possible to do without a scheme of space and time; relativity merely asserts in various ways that its final specification so far eludes our powers that a large number of partial modes of specification can be employed indifferently over a wide range of problems.

On the Variation with Frequency of the Conductivity and Dielectric Constant of Dielectrics for High-Frequency Oscillations.

By G. E. Bairsto, D.Sc., D.Eng.

(Communicated by Prof. J. A. Fleming, F.R.S. Received June 12, 1919.)

1. *Introduction.*

Although we have a certain amount of knowledge regarding the variation of the conductivity of dielectrics with frequency for comparatively low frequencies, within the telephonic range, say, up to 5000 per second, where the conductivity is in general a linear function of the frequency, it cannot be said that any information exists at present as to what happens when we extend the range of frequencies up to those employed in radiotelegraphic work. That energy is dissipated in condensers used in oscillation circuits has been known since 1861, when W. Siemens† pointed out that the glass of a Leyden jar became heated on charge and discharge. Threlfall,‡ extending

* *Cf.* 'Æther and Matter,' chap. xi. (1900). Here in cognate manner the five-dimensional space-time foundation is introduced in order to provide the necessary standards of time and space, which, even though provisional, are indispensable.

† 'Berlin Akad. Monatsber.,' October, 1861.

‡ 'Phys. Rev.,' vol. 4, p. 57, and vol. 5, pp. 21 and 65.

the early experiments of Arno, working with a rotating electrostatic field, found that under these conditions there was no hysteresis loss at $10^7 \sim$ per second in the dielectrics he employed: ebonite, glass, and sulphur. At somewhat lower frequencies of the order of a million · a second, several observers have made measurements of the energy dissipated, and find that condensers have an appreciable decrement. Reference may be made to the following: W. Hahnemann and L. Adelmann,* G. Dupreux,† J. J. Stockley,‡ M. Wien,§ J. A. Fleming and G. B. Dyke,|| L. W. Austin,¶ E. F. W. Alexanderson.**

Most of the measurements were made at working voltages, so it is impossible to say how much of the energy loss is due to brush discharges and how much to a true dielectric conductivity. Moreover, the measurements have generally been confined to some particular frequency. The object of the experiments to be described below was to measure the conductivity of the dielectric over a wide range of frequency, employing continuous oscillations of sine wave form, and of low voltages.

Apart from the importance of the subject in connection with the properties of dielectrics in general, measurements of this kind are of value in another direction, viz., in the theory of the propagation of electromagnetic waves over the earth's surface. Before the theory as developed by Zenneck and others can be applied to practice, we require to know the values of the dielectric constants and conductivities of the constituents of the earth's crust, and the explanation of many anomalous effects which at present are but little understood, would in all probability be cleared up if we were in possession of these data for certain regions of the earth's surface. Loewy,†† with this object in view, has made an extensive series of such measurements, but only with direct currents (D.C.), which we now know gives values of the conductivities which bear no relation to conductivities as

* "Losses in Condensers, and Damping in Wireless Telegraph Circuits," 'Electrotech. Zeit.,' vol. 28, pp. 988 and 1010 (1907).

† "Oscillatory Discharges of Condensers," 'Electrician,' vol. 38, p. 107 (1909); or 'Science Abstracts,' 12 B, No. 906 (1909).

‡ "Effect of Temperature on Damping in Glass Condensers," 'Schweiz. Electrotech. Zeit.,' vol. 6, p. 309 (1909); or 'Science Abstracts,' 12 A, No. 1441 (1909).

§ "Damping in Oscillatory Circuits," 'Ann. der Phys.,' vol. 29, 4, p. 679 (1909); or 'Science Abstracts,' 12 B, No. 1596 (1909).

|| "Energy Losses in High Frequency Circuits," 'Proc. Phys. Soc.,' vol. 23, p. 117 (1911); or 'Science Abstracts,' 14 B, No. 384 (1911).

¶ "Condenser Losses at High Frequencies," 'Journ. Wash. Acad. Sci.,' vol. 1, p. 143 (1911); or 'Science Abstracts,' 15 A, No. 199 (1912).

** "Dielectric Hysteresis at Radio-frequencies," 'Proc. Inst. Radio. Eng.,' vol. 2, p. 137 (1914).

†† 'Ann. der Phys.,' vol. 13, p. 125 (1911).

measured with alternate currents (A.C.). Even at telephonic frequencies the alternate conductivity is much greater than the D.C. conductivity; at radio-telegraphic frequencies the differences become enormous. As a small contribution to the subject, the author has therefore included two typical constituents of the earth's crust in his measurements, viz., slate and marble.

2. *Apparatus and Methods of Measurement.*

The method of measurement adopted is based upon the resonance of a leaky condenser. Two condensers, one the unknown leaky condenser under test, and the other a standard air condenser shunted with a variable high resistance, are alternately placed in series with an inductance, and each adjusted for resonance with a primary source of oscillations, and also for the same maximum current in both. Under these circumstances, the capacities of the two condensers will be the same, and the shunted resistance will be equal to the reciprocal of the conductance for the frequency used during the experiment. Now it is exceedingly difficult to obtain a perfectly *steady** source of undamped alternating currents of high frequency. The condition of equality was therefore determined by means of the specially designed commutator shown in fig. 1. This commutator, driven by an

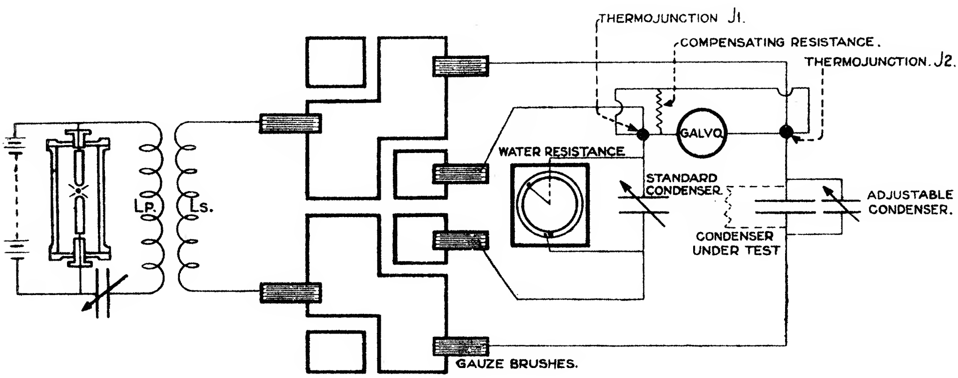

Fig 1. Shewing Commutator & Circuits for High Frequency Measurements.

electric motor, alternately, and for equal times, inserts either of the two condensers in series with the inductance L_s, forming a secondary to the primary L_p. Two thermo-junctions, J_1 and J_2, are arranged, one in each circuit, in such a manner that the E.M.F.'s produced by them oppose one

* As this work was carried out in 1913–1914, three electrode valves of sufficient power were not available.

another. When the galvanometer placed in the circuit common to both gives no deflection, then we know that the two conditions mentioned above are fulfilled. This arrangement being a null reading one, makes it independent of any irregularities in the source of the oscillations. It also enables us to approach zero from either side, for it is apparent that, according as one conductance is greater or less than the other, so will its corresponding resonance current be less or greater than the other, and therefore for one E.M.F. to preponderate over the second.

For ease of construction, the commutator was built up of six brass rings, mounted on red fibre discs supported on a steel spindle. The two end rings as well as the two middle ones were split in half. One half of each split ring was electrically connected to the neighbouring whole ring in the manner shown in fig. 1. Six small gauze brushes lead the current from the various circuits, and in such a manner that once during each revolution of the commutator, and for equal times, each condenser is inserted in the resonating circuit.

The dielectrics were made up into small condensers with tinfoil interleaved between the sheets, each condenser having a capacity of from 300 to 2000 micro-micro-farads. The condenser under test being of a constant value, a series of inductances ranging from 0·005 to 0·1 millihenry were used, depending at what particular frequency measurements were being made.

The adjustable standard of resistance being very high, of the order of half a megohm, a wire resistance was quite out of the question, especially at high frequencies. A water resistance was therefore used. This consisted of a block of paraffin wax, in which a circular channel was cut, about 15 cm. in diameter, the area of channel about 1 sq. cm. A plug of wax was then made in the channel near one terminal (fixed); the other terminal consisted of a small thin plate of copper attached to a radial arm. This enabled one to obtain a continuous range of resistance from zero up to a maximum depending upon circumstances. Either distilled water or slightly acidulated water was poured into the channel, the maximum resistance depending upon the particular dielectric under test, and on the frequency used during the test. The highest resistance obtainable was about 2 megohms. The arrangement which was kept in a glass case was calibrated at the beginning and end of each day's work.

The thermo-junctions were of the type devised by Dr. J. A. Fleming. It is necessary that they should be both of exactly the same resistance in the heater wire, in order that the total resistance decrement of the two circuits should be the same, and what is more, the magnitude of the heater resistance should be as small as possible, consistent with sensitiveness, in order to cut

down the resistance decrement. It may be pointed out that if we are dealing with a constant E.M.F. induced in a resonating circuit having only resistance decrement, the deflection of a thermal ammeter is inversely proportional to the effective resistance of the circuit R. As the effective resistance of the inductance coil L_s was very small, we may take R as being equal to the resistance of the heater wire in the thermo-junction. Then if I be the resonance current, we have

$$I = E/R \text{ (when the circuit is tuned)}$$

$$\text{and heat developed} = RI^2,$$

$$\therefore \quad \text{heat developed} \propto 1/R,$$

$$\therefore \quad \text{galvanometer deflection} \propto 1/R.$$

Hence the smaller we make R, the greater will be the deflection. There are, however, two factors which limit the above. R cannot be made too small, for this means a thick wire and therefore a larger surface for radiation and a lower temperature of the thermo-junction. Again, a small R means a large voltage across the condenser, and this we decided must be kept low in order to prevent any possibility of brush discharge with the thin dielectrics used in making up the condensers.

In the above we have neglected the decrement of the condenser itself, but taking this into account, it would still remain true that the smaller R is the greater will be the galvanometer deflection. It therefore suggests that we should make the heater of the metal having the best electrical conductivity, viz., silver. The heaters finally used were 0·025 mm. diameter, and had a resistance of 0·23 ohm. So, at the highest frequency used, 2×10^6, and a capacity of 1000 micro-micro-farads, the power factor of the inductance and heater would be

$$= R/L_p = pCR = (2\pi \times 2 \times 10^6) \times 1000 \times 10^{-12} \times 0{\cdot}23 = 0{\cdot}0029.$$

The total power factor of the entire oscillating circuit, including the commutator, and apart from the condenser itself, was approximately twice this value, about 0·007. Since this is much less than the power factors of the condensers tested, it follows that the greater part of the decrement of the oscillating circuits resided in the condensers themselves, and the experiments were therefore conducted under the best possible conditions as regards the production of resonance.

The junctions gave with a Paul 10-ohm galvanometer a maximum deflection for 0·08 ampère. Under the above circumstances the E.M.F. on the condenser would be 8 volts. At the lowest frequencies, from half-a-million downwards, in order not to increase this voltage above about 10 volts, a second

pair of junctions were used, having each a resistance of about 0·95 ohm, giving a resistance decrement of approximately the same order of magnitude as before, because we should be using a proportionately larger inductance.

A second requirement of the thermojunctions is that they should be exactly equal as regards sensitiveness, *i.e.*, must give equal deflections on the galvanometer for equal currents in either. With a little manipulation of the soldered junction it is possible to get them so to within 4 or 5 per cent. The remaining difference can be compensated for by shunting the more sensitive one with a resistance until they give the same deflection (see fig. 1).

Although it is possible to obtain no reading on the galvanometer with the commutator running, and definite equal currents in each circuit, it does not follow that there will be a balance for all equal currents, either below or above. This necessarily follows from the impossibility of obtaining both the heating and radiation constants of the two thermo-junctions exactly similar at all temperatures. Fig. 2 shows how close we may get to perfect equality under all conditions. The abscissæ give the current in either of the thermo-

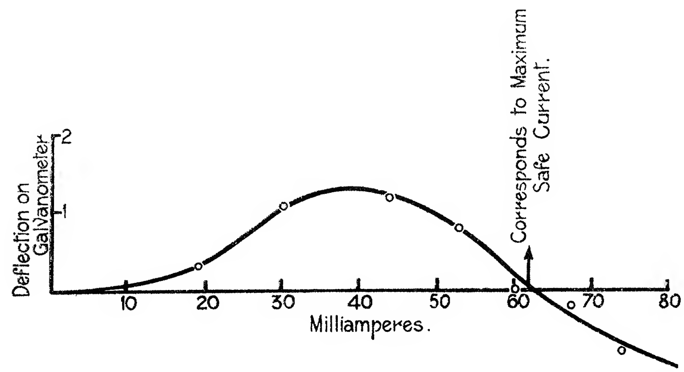

Fig 2. Shewing the Out-of-Balance Deflection on Commutator for Different Currents in the Thermojunctions.

junctions, and the ordinates the out-of-balance deflection on the galvanometer. It will be seen that we have a zero deflection for 63 milliampères, which was the maximum safe current in the junction in question. This corresponds to a galvanometer deflection of 100 divisions. At 40 milliampères we have a maximum out-of-balance current of 1·3 divisions; this, however, is very small compared with the total deflection, and only introduces a possible error of between 1 and 2 per cent. The junctions could stand an overload current up to 80 milliampères.

As the source of oscillations a graphite arc in air, shunted with inductance and capacity, was employed. Above about 200,000 $\sim$ per second, it was

found impossible to obtain sufficient power in the secondary circuit without making the coupling too tight. Placing the arc in compressed air at about five atmospheres pressure enabled a much greater power to be taken out and increased the available frequency to about a million or so. Hydrogen was also tried as recommended by Poulson and others, but while the oscillations are stronger, they are much less steady, and, what is much more unsatisfactory, it gives a very impure wave, and the circuit is exceedingly difficult to tune, as the frequency varies with the arc current. The latter point has also been observed by L. W. Austin.* The graphite arc in compressed air, on the other hand, for some reason which is not very plain, tends to suppress all the higher harmonic components of the main oscillations and gives a nearly pure sine wave.

The limit at which one can obtain steady oscillations with the Duddell arc is about 500,000 $\sim$ per second. Beyond this the arc behaves very erratically and constantly fluctuates ; at 2,000,000 $\sim$ oscillations are only intermittent. The author has succeeded in obtaining fairly steady continuous oscillations up to about 5×10^6 $\sim$ per second by means of the arrangement of circuits shown in fig. 3. The circuit A is tuned to a frequency at which steady oscillations can be obtained, *i.e.*, from 500,000 to 10^6, and circuit B, placed so that it can

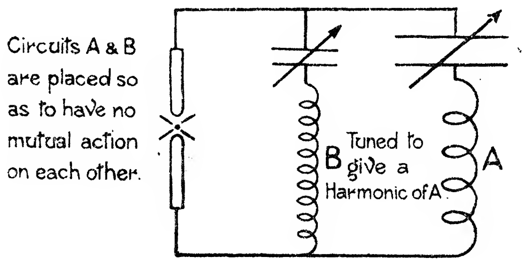

Fig 3. Scheme of Parallel Circuits to give Very High Frequency Currents.

have no mutual action on A, is tuned to a harmonic, either second, third fourth, or fifth of circuit B. Under these circumstances, oscillatory currents of these different frequencies can easily be obtained, circuit A acting with a steadying influence. If circuit A is tuned to a frequency of 10^6 and B to 3×10^6, and the oscillations started, it is found that if the lower frequency circuit A is cut out, the oscillations are immediately stopped in the higher frequency circuit B. On making circuit again, the oscillations in B

* 'Bull. Bureau Standards,' vol. 3, p. 325 (1907).

immediately restart. It was only by the use of this arrangement that it was possible to reach the extreme limit of $3 \times 10^6 \sim$ per second, to which the observations were carried for the last three dielectrics mentioned below.

The actual method of procedure followed in making the measurements was as follows:—With the commutator at rest each condenser was adjusted for maximum resonance current. The commutator was then set running and the out-of-balance current flowing through the galvanometer was brought to zero by varying the resistance shunted across the standard air condenser. This adjustment in the case of the poorer dielectrics would need a slight alteration in capacity of the standard air condenser to bring the circuit into resonance again. A final readjustment of the shunted resistance would then bring the circuits into perfect equality.

If C and S be the capacity and conductance of the unknown condenser under test, C_c the capacity in parallel with it to produce resonance, C_s the capacity in the standard circuit to produce resonance, and R the shunted resistance, then

$$C = C_s - C_c \qquad \text{and} \qquad S = 1/R.$$

From C and S knowing the ratio of area to thickness for the dielectric we can calculate σ and K the specific alternate current conductivity, and the dielectric constant.

The frequency from 400,000 $\sim$ per second and upwards was measured with a Fleming cymometer. The lower range was determined with another made-up cymometer, consisting of a variable air condenser, reading from 200 and 3300 micro-micro-farads combined with a single layer solenoidal coil. The inductance of the latter was calculated by Russell's formula and found to be 0·238 millihenry; its measured value at telephonic frequencies by the Anderson bridge method was found to be equal to 0·233 millihenry. This value requires a slight correction when used at high frequencies. This correction was applied by Heaviside's formula:*

$$L_0 - L_\infty = 0{\cdot}026 \left(\frac{N^2 ad}{l}\right) \text{ microhenry},$$

where N equals number of turns, a equals radius of coil, d equals diameter of wire and l equals the length of the coil. These quantities were respectively 66, 4·15, 0·091 and 7·9, and therefore gave a correction of 0·0055 millihenry. The value of the coupling inductance at high frequencies was therefore taken as being equal to 0·233 − 0·005 or 0·228 millihenry.

* 'Electrical Papers,' vol. 1, p. 356.

3. *Discussion of Experimental Results.*

In the Tables are given the experimental results obtained for the following dielectrics:—Blotting paper, glass, guttapercha, vulcanised indiarubber, marble and slate. The general nature of the results is to show that the conductivity increases enormously as the frequency increases. The linear law found by Dr. Fleming and Mr. Dyke* connecting the two quantities at telephonic frequencies is now no longer obeyed, but the conductivity gradually tends to a maximum and then seems to decrease again.

We will now discuss the behaviour of each particular dielectric in turn.

(a) *Blotting Paper.*—Table I gives the results for a condenser made up of good white blotting paper which had been previously dried in an oven. The first three columns give the values of the measured conductance S and capacity C at given frequencies. The last three columns give the values of the dielectric constant, specific conductivity, and S/Cp (approximately proportional to the power-factor) as calculated from these quantities S and C. The observations are depicted in fig. 4. It will be seen that the conductivity at first proportional to the frequency quickly rises to a maximum at about 600,000 ~ per second, and then decreases again. In the curve showing the variation of dielectric constant with frequency we have a sharp fall at

Table I.—Dielectric: Dried Blotting Paper. Area = 105 sq. cm.

Thickness = 0·042 cm. $\dfrac{\text{Area}}{\text{thickness}} = 2480.$

Frequency, ~ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/Cp.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube.
920	400	0·875	0·038 (0·009)†	1·83	35·2
2,760	395	0·135	0·019 (0·0095)†	1·81	53
4,600	393	0·190	0·0165 (0·0105)†	1·80	76
180,000	375	8·25	0·019	1·72	3,320
210,000	380	10·4	0·021	1·74	4,200
235,000	377	16·6	0·030	1·72	6,700
285,500	375	19·5	0·029	1·72	7,900
320,000	375	23·0	0·030	1·72	9,250
360,000	375	25·5	0·030	1·72	10,300
445,000	375	50·5	0·048	1·72	20,200
495,000	373	62·5	0·054	1·71	25,200
580,000	370	68·5	0·051	1·69	27,400
790,000	350	52·5	0·0305	1·60	21,200
880,000	340	42·5	0·0195	1·55	17,100

† The numbers in the brackets are the values of the power-factor calculated by subtracting the ordinary direct current component from the total conductance.

* 'Journ. Inst. Elec. Eng.,' vol. 49, p. 323 (1912).

telephonic frequencies and then a gradual decrease as the frequency increases. In drawing the S/Cp curve we have, for the telephonic range of frequencies,

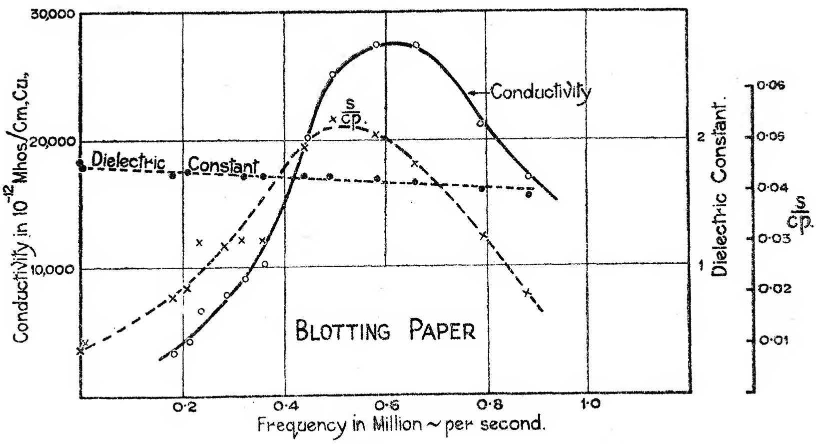

Fig 4. Variation with Frequency of the Conductivity, S/cp, & Dielectric Constant of Blotting Paper.

separated out the ordinary direct current component of the conductivity which is superposed on the true alternate current conductivity. Thus if $S =$ total measured conductivity, and $S_o =$ that due to the direct current component, then $(S-S_o)/Cp$ represents the power-factor due to the alternate current conductivity alone. These values are given in the brackets in column 4. At the higher frequencies S_o is too small compared with S to have any influence on S/Cp. The result of these calculations is to give an intercept on the axis of conductivity indicating that from $n = 0$ up to about $n = 50{,}000$ ~ per second we have a constant power-factor (see fig. 4) approximately equal to 0·01. Above this there is an increase in the power-factor proportional to the frequency, that is to say, S/Cp may be represented as the sum of two quantities:—

$$S/Cp = x + yp,$$

or, what is the same thing,

$$\sigma = an + bn^2.$$

It is apparent, therefore, that we are dealing with two separate effects, one implying a loss proportional to the frequency, and the other a loss proportional to the square of the frequency. At telephonic frequencies the first term is the only one of importance. At higher frequencies the second has the more influence. The first implies a loss which is independent of the time taken for taking the dielectric through a complete cycle, and we will, therefore, tentatively designate it a "dielectric hysteresis" loss.

The second which is a loss depending upon the time taken to complete the cycle will, for reasons to be given later, be called a "viscous" loss.

A comparison of the different quantities for low and for high frequencies is instructive. Thus, the maximum power-factor (0·054) is $5\frac{1}{2}$ times as great as the power-factor at telephonic frequencies (0·01). The calculated value of the D.C. component is $26 \cdot 7 \times 10^{-12}$ mhos per cubic centimetre. Therefore the maximum conductivity at high frequencies is 27,500/26·7, or 1020 times as great as the conductivity with steady currents.

(b) *Crown Glass.*—The condenser used for these experiments was the thin crown-glass used for the cover-glasses of microscope slides. The results are given in Table II and are plotted in fig. 5. The general nature of the curves is similar to that of the ones for blotting-paper. The linear part of the conductivity curve extends over a wider range, and the point of maximum conductivity is a little lower, viz., 510,000 ~ per second. The calculated value of the D.C. conductivity is 12×10^{-12}, and therefore the maximum alternating current conductivity is 30,500/12, or 2500 times as large.

The S/Cp curve is slightly different. It will be seen that it has a relatively larger intercept on the axis of co-ordinates, implying that hysteresis contributes more to the conductivity than viscosity, for the maximum S/Cp (0·023) is only 50 per cent. greater than the constant S/Cp corresponding

Table II.—Dielectric: Crown-glass. Area = 55 sq. cm.

Thickness = 0·206 cm. $\dfrac{\text{Area}}{\text{thickness}} = 2760.$

Frequency, ~ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/Cp.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube
920	1547	0·16	0·018 (0·0145)*	6·60	61
2,760	1535	0·435	0·017 (0·0155)*	6·56	164
4,600	1530	0·682	0·0145 (0·0145)*	6·53	256
150,000	1505	25·2	0·0175	6·40	9,450
180,000	1500	30·0	0·0175	6·35	11,200
230,000	1490	41·5	0·019	6·30	15,600
270,000	1475	58·0	0·023	6·25	21,800
295,000	1470	60·0	0·022	6·2	22,500
335,000	1470	68·0	0·022	6·2	25,500
395,000	1475	70·5	0·019	6·25	26,500
460,000	1470	81·5	0·019	6·2	30,500
510,000	1455	80·0	0·017	6·15	30,000
580,000	1470	74·5	0·014	6·2	28,000
620,000	1460	65·5	0·0115	6·15	24,500
705,000	1460	61·5	0·0095	6·15	23,000
800,000	1470	61·5	0·0085	6·2	23,000

* The numbers in the brackets are the values of the power-factor calculated by subtracting the ordinary direct current component from the total conductance.

to low frequencies (0·015). The further course of the curve is interesting, for it eventually falls to a value only half that of the intercept for $n = o$, and

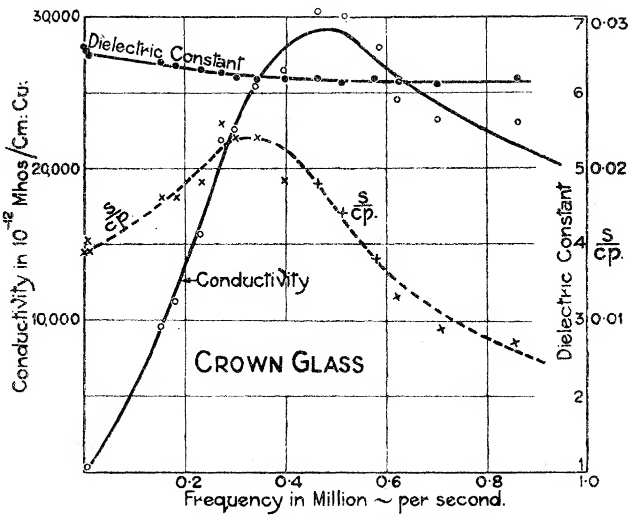

Fig 5. Variation with Frequency of the Conductivity, s/cp, & Dielectric Constant of Crown Glass.

is still rapidly decreasing. This indicates that the hysteresis loss per cycle which is constant at low frequencies, decreases and tends to become zero at higher frequencies, viscosity only being of importance.

During the observations on glass, means were taken to see whether the alternate current conductivity varied with the voltage.* This was done by balancing the commutator for some particular coupling of the primary and secondary and then altering it by either increasing or decreasing the distance between them. No alteration of the balance was required although the voltage conditions were such that at the lower limit only about 2 volts were across the condenser, and at the higher limit a perceptible noise inside the condenser indicated that brushing had just set in. This confirms for high frequencies what B. Monasch found at telephonic frequencies—the energy loss in a dielectric is strictly proportional to the square of the voltage, provided brush discharge is eliminated.*

(c) *Vulcanised Indiarubber.*—This dielectric is one of the two (gutta-percha being the other) out of all those tested by Dr. Fleming and Mr. Dyke, whose conductivity they found was incapable of being represented within the telephonic range of frequency by the linear law

* "Dielectric Losses," 'Ann. der Phys.,' vol. 22, p. 905 (1907); or 'Science Abstracts,' 10 B, No. 897 (1907).

$\sigma = a + bn$, σ increasing much more rapidly. This is illustrated by the first three rows of figures in Table III; the power-factor for $n = 920$ is 0·002, whereas for $n = 4600$ it is nearly twice as great, being 0·0035. The reason for this is clearly seen on inspection of fig. 6, for there we see that

Table III.—Dielectric: Vulcanised Indiarubber. Area = 55 sq. cm.

$$\text{Thickness} = 0{\cdot}053 \text{ cm.} \quad \frac{\text{Area}}{\text{thickness}} = 1030.$$

Frequency, ~ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/Cp.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube.
920	249	0·003	0·002	2·73	3
2,760	247	0·012	0·0025	2·70	12
4,600	247	0·023	0·0035	2·70	22
210,000	245	8·2	0·025	2·68	7,900
235,000	243	9·50	0·027	2·65	9,100
305,000	243	16·3	0·035	2·65	15,900
325,000	245	18·8	0·038	2·68	18,200
360,000	245	17·4	0·032	2·68	17,000
375,000	240	15·9	0·028	2·62	15,500
395,000	240	16·1	0·027	2·62	15,700
405,000	230	18·2	0·031	2·51	17,600
430,000	235	26·3	0·0415	2·57	25,500
480,000	235	38·0	0·053	2·57	37,000
515,000	225	42·5	0·058	2·45	41,000
560,000	230	50·0	0·062	2·52	48,500
650,000	240	48·5	0·049	2·62	47,000
720,000	240	39·3	0·036	2·62	38,000
880,000	225	29·5	0·0235	2·45	28,800

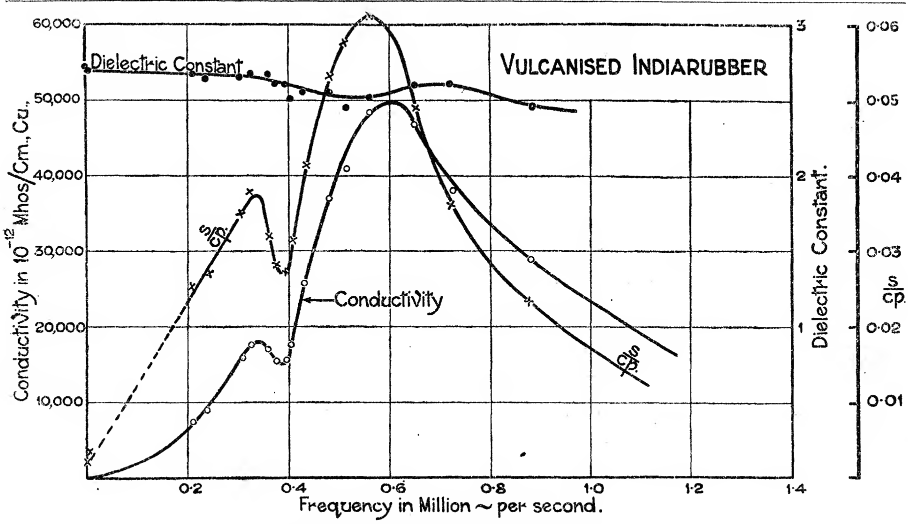

Fig 6. Variation with Frequency of the Conductivity, S/cp, & Dielectric Constant of Vulcanised Indiarubber.

the intercept on the co-ordinate axis is very small, so that it is only over a very short range, say up to 400 $\sim$ or 500 $\sim$ per second, that S/Cp can be called even approximately constant (by interpolation being about 0·0015). The viscous loss is, in fact, of so much greater importance, that at 350,000 $\sim$ per second, the power-factor is 0·038, or 23 times as large as at very low frequencies. Hysteresis is therefore, comparatively speaking, absent.

For a very considerable range, from $n = 0$ up to $n = 350{,}000$ $\sim$ per second, S/C_p is exactly a linear function of the frequency, or

$$\sigma = bn^2.$$

The conductivity has a maximum value at about 560,000 $\sim$ per second, and increases so rapidly with frequency that it is 16,000 times as great as at $n = 1000$. There is also a subsidiary maximum at 350,000 $\sim$ per second, causing a slight hump on the curve, and indicating the existence of two components in the dielectric.

It remains now to point out a remarkable agreement between the maximum power-factor observed above on the power-factor-frequency curve and the maximum power-factor observed by Fleming and Dyke for vulcanised indiarubber at certain low temperatures. They found that at about $-30°$ C. the conductivity at a given frequency rose very rapidly and reached a maximum of considerable value, fourteen times that at ordinary temperatures. The power-factor at this point was 0·034, and was independent of the frequency. Now, the power-factor observed above for the first hump on the conductivity-frequency curve is 0·038, which agrees fairly well with the former figure. If we allow for the slight change of capacity with frequency and temperature, the agreement is still closer.

(d) *Guttapercha.*—This dielectric is another material that does not follow a linear law in the variation of conductivity with frequency within the telephonic range. In the last case (vulcanised indiarubber), it was due to the fact that the hysteresis loss was negligible in comparison with the viscous loss. In the case of guttapercha, however, the divergency results from quite a different cause, being due to the presence of a component in the dielectric having a frequency of maximum conductivity which is only just outside the telephonic range. Thus, if we inspect Table IV, we see that the power-factor increases from 0·014 at $n = 920$ $\sim$ per second up to 0·027 at 15,000 $\sim$ per second. This was the upper limit imposed by the limits of audition on measurements made with the bridge and telephone. At $n = 210{,}000$ S/Cp, however, has fallen to 0·019, implying the existence of a slight hump on the conductivity-frequency curve at 50,000 $\sim$ per second. It was not possible to make any measurements at lower frequencies, because with the present experimental arrangements they would entail a considerable

Table IV.—Dielectric: Guttapercha. Area = 110 sq. cm.

Thickness = 0·0582 cm. $\dfrac{\text{Area}}{\text{thickness}} = 1890.$

Frequency, ~ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/C*p*.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube.
920	485	0·38	0·014	2·92	20
2,760	479	0·158	0·019	2·88	84
4,600	475	0·305	0·022	2·85	160
15,000	470	0·79	0·027	2·82	416
180,000	460	11·2	0·019	2·76	5,900
235,000	460	11·8	0·018	2·76	6,250
250,000	455	13·4	0·018	2·73	7,100
295,000	455	15·0	0·018	2·73	7,950
320,000	460	17·9	0·019	2·76	9,500
365,000	455	19·6	0·0195	2·73	10,300
400,000	453	22·3	0·020	2·72	11,800
410,000	450	23·4	0·021	2·70	12,400
445,000	453	29·1	0·021	2·72	15,400
555,000	450	45·3	0·028	2·70	24,000
600,000	450	61·4	0·036	2·70	32,500
690,000	450	70	0·036	2·70	37,000
1,000,000	445	52	0·019	2·67	27,500
1,040,000	440	47	0·0165	2·64	24,800
1,350,000	440	39·5	0·0105	2·64	20,800
1,700,000	435	27·8	0·006	2·62	14,700

voltage on the condenser to obtain sufficient current. For this reason, the curve has had to be filled in tentatively by means of the dotted line. From about 300,000 ~ per second and upwards, the conductivity and S/C*p* curves (see fig. 7) are precisely similar to those of the dielectrics already considered,

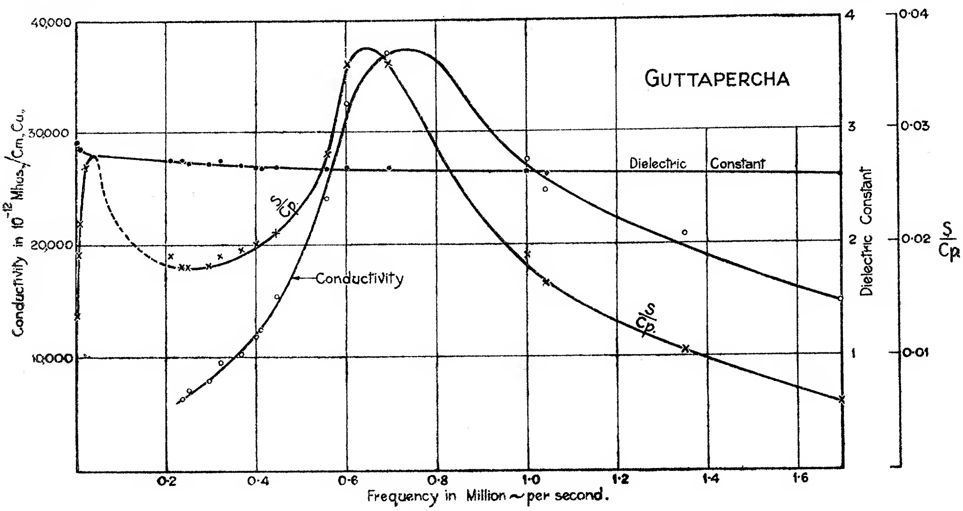

Fig 7. Variation with Frequency of the Conductivity, S/Cp, & Dielectric Constant of Guttapercha.

in that they rapidly rise up to a maximum and decrease again; the frequency for maximum conductivity in this case is about 750,000 $\sim$ per second.

By interpolation the power-factor for very low frequencies ($n = 0$) is 0·011, and it will be seen that if we prolong the base of the second hump of the S/Cp curve backwards in a straight line it will also cut the vertical axis at a point very close to this. In other words, each of the two components in the dielectrics, except for frequencies in the neighbourhood of its own peak, has little influence on the conductivity at other parts of the curve, both commencing with a common S/Cp arising from hysteresis.

As in the previous case the maximum power-factor observed at the first hump on the power-factor curve agrees approximately with the maximum power-factor observed at telephonic frequencies by Fleming and Dyke at a certain low temperature (8° C.). They found a maximum power-factor at this temperature, which was *independent* of frequency and amounted to 0·025. The maximum power-factor observed above on the conductivity-frequency curve at ordinary temperatures is 0·028. This coincidence is very striking and may be of interest in developing the theory of dielectrics under alternating E.M.Fs.

(e) *Marble.*—The next two dielectrics to be considered are typical constituents of the earth's crust. In the case of marble the curves present some points of difference from those already considered. The experimental results are given in Table V and depicted in fig. 8. They refer to a condenser made up of 20 square marble slabs* about 0·7 cm. thick, and effective area coated with tinfoil of 2400 sq. cm. Only 16 were effective, the remaining 4 being used on the outside of the condenser to carry the lines of force from the outside faces of the last tinfoil electrode. The whole was bolted together with two bolts.

As with the other dielectrics, the dielectric constant at first falls rapidly within the telephonic range, and then slowly decreases as the frequency reaches those used in radiotelegraphy. The conductivity-frequency curve however is different, in that after the usual linear stage at the beginning, the curve in approaching its maximum gradually bends over, instead of turning upward. The result of this is that the power-factor throughout the whole range decreases with increase of frequency.

At the very highest frequencies it may be noticed that, as regards power-factor, marble is better than any of the other dielectrics, the lowest S/Cp reached being 0·0033.

The calculated value of the D.C. conductivity is 310×10^{-12} mhos per

* I have to express my indebtedness to Dr. J. A. Fleming, F.R.S., for placing these marble slabs at my disposal.

cubic centimetre, while the maximum A.C. conductivity is 23,000, the ratio being 74.

Table V.—Dielectric: Marble. Area = 2400 sq. cm.

Mean thickness = 0·67 cm. $\dfrac{\text{Area}}{\text{thickness}} = 3500.$

Frequency, ~ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/C*p*.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube.
920	2771	1·48	0·093 (0·025)*	8·88	425
2,760	2707	2·12	0·047 (0·024)*	8·75	627
4,600	2675	2·58	0·035 (0·021)*	8·55	768
44,000	2570	13·7	0·019	8·40	3,950
156,000	2545	33	0·0135	8·20	9,500
235,000	2500	40	0·011	8·05	11,500
285,000	2510	54	0·012	8·1	15,500
320,000	2510	56	0·011	8·1	16,000
390,000	2500	59·5	0·0095	8·05	17,000
450,000	2440	63	0·0088	7·85	18,000
610,000	2440	57	0·006	7·85	20,000
740,000	2440	60	0·0053	7·8	21,000
930,000	2405	63	0·0045	7·75	22,000
1,050,000	2320	63	0·0041	7·50	22,000
1,170,000	2300	61·5	0·0036	7·40	21,500
1,400,000	2265	65·5	0·0033	7·30	23,000

* The numbers in the brackets are the values of the power-factor calculated by subtracting the ordinary direct current component from the total conductance.

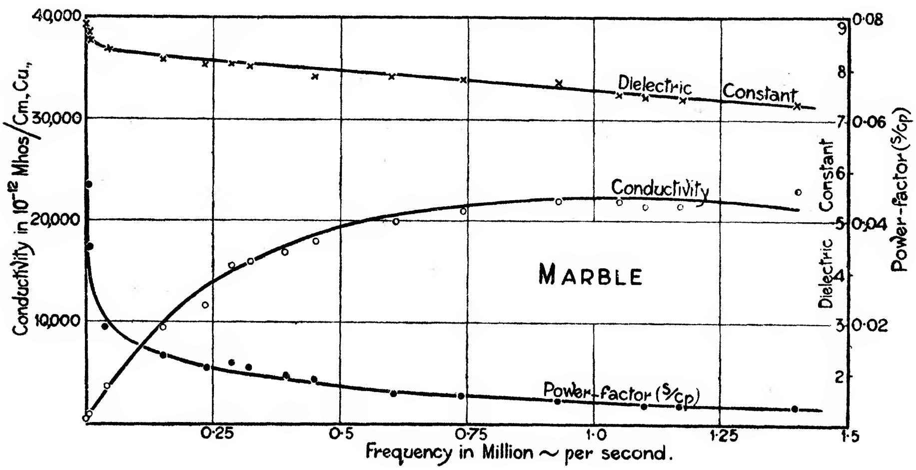

Fig 8. Variation with Frequency of the Conductivity, S/Cp, & Dielectric Constant of Marble.

(f) *Slate.*—The results for the slate condenser are given in Table VI and depicted in fig. 9. The frequency was taken up to 3,000,000 cycles per

second, and it will be seen that the conductivity is still rising, although there seems to be a tendency for the curve to flatten and turn over again. At the same time there are several irregularities on the curve, which seem to indicate that there are several component materials in the slate. The most noticeable point about the dielectric constant curve is its enormous drop at low frequencies, K being about 50 at $n = 920$, whereas at 500,000 $\sim$ per second, it has fallen to about 10.

This large drop in K causes the power-factor to be approximately constant over a wide range of frequency, whereas if we calculate $(\sigma - \sigma_0)/n$, which is proportional to the amount of work done per cycle due to the alternating conductivity, it will be seen that this quantity after being approximately

Table VI.—Dielectric: Slate. Area = 55·8 sq. cm.

Thickness = 0·162 cm. $\dfrac{\text{Area}}{\text{thickness}} = 344.$

Frequency, $\sim$ per sec.	Capacity in micro-micro-farads C.	Conductance in micro-mhos S.	S/Cp.	$(\sigma-\sigma_0)/n \times 10^6$.	Dielectric constant.	Conductivity in micro-micro-mhos per cm. cube σ.
920	1520	3·00	0·342	7·8	50·0	0·00872
2,760	1255	8·05	0·369	7·9	41·5	0·0233
4,600	1145	13·3	0·400	8·0	37·5	0·0386
12,000	850	29·5	0·455	7·2	28·0	0·086
44,000	620	69	0·405	5·95	20·5	0·20
110,000	500	115	0·33	3·1	16·5	0·34
230,000	450	190	0·29	2·4	14·8	0·55
300,000	390	250	0·34	2·4	12·8	0·73
410,000	390	292	0·295	2·05	12·9	0·85
520,000	350	355	0·255	2·0	11·5	1·03
620,000	315	400	0·250	1·85	10·3	1·15
700,000	320	415	0·245	1·7	10·5	1·20
780,000	325	455	0·28	1·7	10·6	1·32
830,000	325	480	0·28	1·7	10·6	1·40
880,000	330	500	0·27	1·65	10·8	1·45
940,000	340	525	0·26	1·65	11·2	1·53
980,000	320	525	0·265	1·6	10·6	1·53
1,020,000	310	530	0·265	1·5	10·3	1·54
1,050,000	340	525	0·25	1·45	11·2	1·53
1,150,000	285	595	0·28	1·5	9·3	1·72
1,190,000	290	630	0·29	1·5	9·6	1·83
1,240,000	275	625	0·245	1·45	9·15	1·82
1,340,000	305	665	0·26	1·45	10·1	1·94
1,390,000	305	640	0·24	1·35	10·1	1·86
1,450,000	305	650	0·235	1·3	10·1	1·89
1,520,000	290	665	0·24	1·35	9·5	1·93
1,650,000	275	725	0·25	1·3	9·10	2·11
1,760,000	270	770	0·26	1·3	8·95	2·25
1,850,000	270	810	0·255	1·25	8·95	2·36
1,900,000	270	840	0·26	1·3	8·95	2·45
2,170,000	275	860	0·26	1·15	9·1	2·5
2,650,000	270	860	0·19	0·95	8·95	2·5

constant within the telephonic range decreases with increase of frequency from about $7{\cdot}8 \times 10^{-12}$ mhos per cycle per second to about 1×10^{-12}.

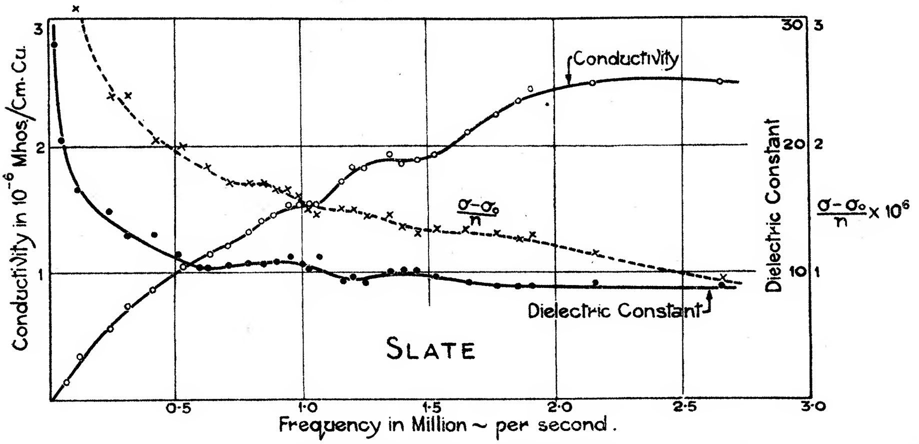

Fig 9. Variation with Frequency of the Conductivity, $\frac{\sigma-\sigma_0}{n}$, & Dielectric Constant of Slate.

The maximum A.C. conductivity is $2{\cdot}5 \times 10^{-6}$ mhos per cubic centimetre, and the calculated value of the D.C. conductivity is 0·0015, hence the ratio of the two quantities is 1700.

Summary.

1. Measurements of the conductivity and dielectric constants of dry blotting paper, glass, vulcanised indiarubber, guttapercha, marble, and slate are given for alternating currents of low voltage and sine wave form, and for a wide range of high frequencies.

2. It is shown in all cases that there are present two independent sources of loss. One, a hysteresis loss, is usually the only one that is important at telephonic frequencies. The second, a viscous loss, has more influence at higher frequencies. The first loss is one which is independent of the time taken for a complete cycle, while the second depends upon this time and at low frequencies gives rise to a loss which varies as the square of the frequency.

3. The linear law found at telephonic frequencies connecting σ with frequency, is no longer obeyed at high frequencies, but σ gradually rises to a maximum, and then decreases again. This rise to a maximum is very rapid for some substances, *e.g.*, guttapercha and vulcanised indiarubber, while with slate and marble, the maximum is approached by the curve bending over instead of turning up. This leads to a very flat curve for which σ is practically constant over a considerable range of frequency.

4. The maximum A.C. conductivity is very much greater than the D.C.

conductivity. In the case of glass, it is 2500 times as large, while in the case of vulcanised indiarubber, the maximum A.C. value is even 16,000 times as large as the A.C. value at $n = 1000 \sim$ per second.

5. The constant hysteresis loss per cycle at low frequencies tends to become zero at very high frequencies.

6. The dielectric constant after a rapid drop at low frequencies, slowly decreases as the frequency rises.

In conclusion, I wish to express my indebtedness to Dr. J. A. Fleming, F.R.S., for placing the resources of his laboratory at my disposal, and for much valuable advice during the course of the work which was carried out during the years 1913 and 1914.

On the Secondary Spectrum of Hydrogen.

By T. R. Merton, M.A., D.Sc.

(Communicated by Prof. A. Fowler, F.R.S. Received August 15, 1919.)

[Plates 6 and 7.]

Theoretical investigations of the origin of spectra in relation to the structure of the atom have concentrated especially on the spectrum of hydrogen, on account of the supposed simplicity of the hydrogen atom. They have, however, been confined almost exclusively to the Balmer series, and have ignored the difficulties which arise from the fact that hydrogen possesses another spectrum, usually known as the secondary spectrum, which is of great complexity, and the co-ordination of whose lines into recognised bands or series of lines is still in a very unfinished state. The investigations of Buisson and Fabry,* in which the physical widths of spectrum lines were measured with the interferometer, refer at least a part of the lines of the secondary spectrum to the hydrogen atom, and the complications introduced into theoretical investigations cannot therefore be impartially waived by the assumption that the molecule is concerned in the production of the secondary spectrum.

As regards the relation of the two spectra, there is abundant evidence of a fundamental difference in their origin. In many celestial spectra the lines of the Balmer series constitute one of the most prominent features, whilst the identification of lines of the secondary spectrum is at least extremely

* 'Journal de Physique,' vol. 2, p. 442 (1912).